

Interaction corrections: temperature and parallel field dependencies of the Lorentz number in two-dimensional disordered metals.

G. Catelani

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853

(Dated: February 6, 2008)

The electron-electron interaction corrections to the transport coefficients are calculated for a two-dimensional disordered metal in a parallel magnetic field via the quantum kinetic equation approach. For the thermal transport, three regimes (diffusive, quasiballistic and truly ballistic) can be identified as the temperature increases. For the diffusive and quasiballistic regimes, the Lorentz number dependence on the temperature and on the magnetic field is studied. The electron-electron interactions induce deviations from the Wiedemann-Franz law, whose sign depend on the temperature: at low temperatures the long-range part of the Coulomb interaction gives a positive correction, while at higher temperature the inelastic collisions dominate the negative correction. By applying a parallel field, the Lorentz number becomes a non-monotonic function of field and temperature for all values of the Fermi-liquid interaction parameter in the diffusive regime, while in the quasiballistic case this is true only sufficiently far from the Stoner instability.

PACS numbers: 71.10.Ay, 72.15.Gd, 72.15.Eb

I. INTRODUCTION

A standard result of the Drude-like theory of transport in disordered metals is the Wiedemann-Franz law¹ relating the (Drude) thermal (κ_D) and electrical (σ_D) conductivities via the Lorentz number L_0 :

$$\frac{\kappa_D}{\sigma_D T} = L_0 \equiv \frac{\pi^2}{3e^2}, \quad (1.1)$$

where T is the temperature in energy units ($k_B = 1$) and e is the electronic charge. “Drude-like theory” means that two assumptions are made in order to calculate the transport coefficients: 1. the electrons do not interact with each other; 2. the scattering of the electrons onto the impurities is elastic.^{2,3} While it was shown long ago⁴ that the interplay of electron-electron interactions and disorder leads to logarithmically divergent, temperature-dependent corrections to the electrical conductivity at low temperatures $T \ll \hbar/\tau$ (τ is the mean free time for the impurity scattering), it is only recently that such effects have been correctly evaluated at higher temperatures⁵ and for the thermal transport.^{6–8} In particular early calculations^{9,10} of the interaction corrections to the thermal conductivity arrived at contradictory results, due to technical difficulties in the proper construction of the energy current density operator (both in the diagrammatic technique and in the kinetic equation approach). This issue has been resolved in Ref. 6, where the local form of the collision integral for the kinetic equation is also presented.

In deriving the quantum kinetic equation, a proper description of the disordered Fermi-liquid is obtained by introducing bosonic soft modes (interacting electron-hole pairs) which contribute to the energy transport but, being neutral, not to the charge transport. For interaction in the triplet channel these bosons have a total spin $L = 1$; this spin degree of freedom is affected by the magnetic field: the description of such effects is a central part

of the present work. By extending the results of Ref. 6, I analyze in detail, for two-dimensional systems, the temperature and parallel magnetic field H dependencies of the “generalized” Lorentz number L , defined as

$$L(T, H) \equiv \kappa(T, H)/\sigma(T, H)T, \quad (1.2)$$

where, due to the electron-electron interaction corrections, both conductivities are temperature- and field-dependent (a similar analysis for zero-dimensional systems – open quantum dots – is presented in Ref. 11; the parallel field dependence of σ is considered in Ref. 12). Because of difficulties in accurately measuring the electronic thermal conductivity, very few experiments have been performed in two-dimensional systems with regards to the thermal transport – for example the Wiedemann-Franz law was found to hold¹³ in a Si MOSFET sample within the experimental accuracy, and the validity of this law was checked for the weak localization correction.¹⁴ One of the difficulties in determining κ is in separating the electronic contribution to the total thermal conductivity from the phonons’ contribution; however, by measuring the thermal conductivity in the presence of a magnetic field it may be possible to extract the electronic field-dependent part, as done e.g. for cuprate superconductors.¹⁵ Since new methods for measuring the thermal conductivity in thin films are being explored,¹⁶ the study of the field dependence of L could be experimentally relevant.

The paper is organized as follows: in the next section I examine the temperature dependence of the Lorentz number L to identify different regimes as a function of the dimensionless parameter $T\tau/\hbar$ and to discuss the various approximations involved. In Sec. III I present the results for the dependence of L on the parallel magnetic field. The derivation of these results is given in Sec. IV. After the conclusions, I briefly consider in Appendix A the field-dependent correction to the specific heat. Appendix B contains some mathematical details,

and Appendix C a discussion of the electrical magnetoconductivity in parallel field.

II. TEMPERATURE DEPENDENCE OF THE LORENTZ NUMBER.

This section contains the result for the temperature dependence of the Lorentz number L . It is convenient to separate L into a “Wiedemann-Franz law” part L_0 [Eq. (1.1)] and a “violation” part δL as follows:

$$L(T) = L_0 + \delta L(T), \quad (2.1)$$

and for the correction δL to distinguish the contributions due to interactions in the singlet and in the triplet channels:

$$\delta L = \delta L^s + \delta L^t. \quad (2.2)$$

For clarity, the two terms are considered separately. The results given below are derived in Sec. IV A within the quantum kinetic equation approach – this is a perturbative approach with $1/g$ as the small parameter, where $g = \sigma/(2e^2/h) \gg 1$ is the dimensionless conductance; it assumes the validity of the Fermi-liquid picture, which in turns requires $T \ll E_F$, with E_F the Fermi energy.

A. Singlet channel.

The singlet correction δL^s is, with logarithmic accuracy:

$$\begin{aligned} \frac{\delta L^s}{L_0} = & \frac{1}{\pi g} \left[g_1(2\pi T\tau/\hbar) \ln(r_s E_F/T) \right. \\ & - \frac{1}{4} g_2(\pi T\tau/\hbar) \ln(1 + (\hbar/2\pi T\tau)^2) \\ & \left. - \frac{1}{5} \left(\frac{2\pi T\tau}{\hbar} \right)^2 \ln(E_F/T) \right], \end{aligned} \quad (2.3)$$

where the functions g_1 and g_2 , given in Eqs. (4.4), describe the crossover from the low temperature diffusive regime to the higher temperature quasiballistic one, and r_s is the “gas parameter” characterizing the interaction strength:

$$r_s = \frac{\sqrt{2}e^2}{\varepsilon \hbar v_F} \quad (2.4)$$

with v_F the Fermi velocity and ε the dielectric constant.

In the diffusive regime $T \ll \hbar/2\pi\tau$, both g_1 and g_2 tend to 1 and Eq. (2.3) reduces to:

$$\frac{\delta L_d^s}{L_0} = \frac{1}{2\pi g} \ln \left(\frac{\pi \hbar D k^2}{2T} \right), \quad (2.5)$$

where $D = \tau v_F^2/2$ is the diffusion constant and $k = 2\pi\nu e^2/\varepsilon$ is the 2D inverse screening radius (where $\nu =$

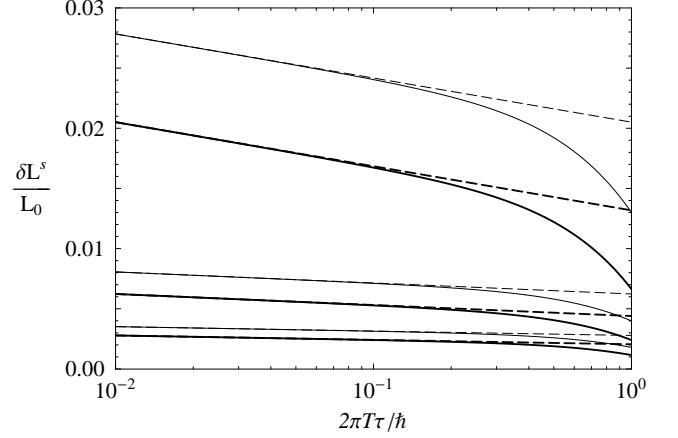


FIG. 1: Relative correction to the Lorentz number as a function of temperature in the diffusive regime. Solid lines are given by Eq. (2.3), while dashed lines by the approximate expression (2.5). For thick lines $r_s = 0.1$ and for thin ones $r_s = 1$; the conductances are for each pair of curves (top to bottom): $g = 100$, $g = 400$ and $g = 1000$.

m/π is the 2D density of states). Compared to the Altshuler-Aronov interaction correction to the electrical conductivity,⁴ this logarithmic correction to the Lorentz number has a completely different physical origin: it arises from the energy transported over long distances by the neutral bosonic soft modes of the interacting electron system.⁶ At low temperatures, this additional channel for the energy transport (as compared to the charge transport) leads to an increase in the thermal conductivity over the electrical conductivity and therefore to an enhancement of the “generalized” Lorentz number (1.2).

In Fig. 1 I plot the relative change of the Lorentz number $\delta L^s/L_0$, Eq. (2.3), as a function of $2\pi T\tau/\hbar$ for three conductances ($g = 100, 400$ and 1000) and for two values of the interaction strength ($r_s = 0.1$ and $r_s = 1$); for comparison the curves obtained using the approximate expression (2.5) are also shown. From the figure and the dependence on r_s in Eq. (2.3) it follows that the deviation from the Wiedemann-Franz law grows with the interaction strength.²⁰ For low conductances and temperatures the (positive) change in the Lorentz number is of the order of a few percent; unfortunately the uncertainty in measurements of the thermal conductivity in metallic films¹⁶ is also of this magnitude, making a comparison with the present theory pointless.

As the temperature increases, the inelastic collisions between the electrons and the bosons tend to inhibit the energy transport more efficiently. The quasiballistic regime is reached in the temperature range

$$\hbar/2\pi\tau \ll T \ll T_{qb}^s, \quad (2.6)$$

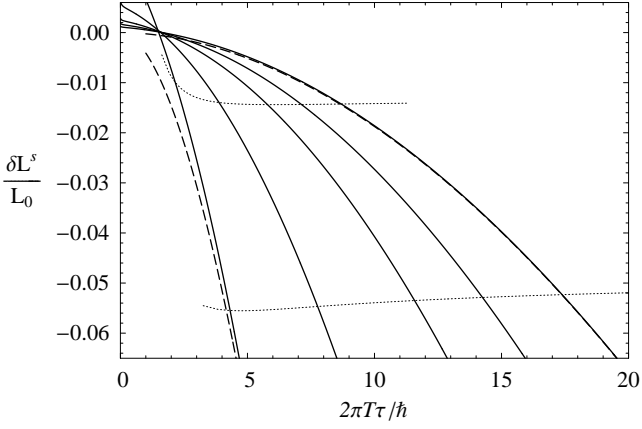


FIG. 2: Relative correction to the Lorentz number as a function of temperature in the quasiballistic regime. Solid lines are given by Eq. (2.3) with $r_s = 0.1$, while dashed lines by the approximate expression (2.9). The conductances are (left to right): $g = 100, 400, 1000, 1600$ and 2500 . Thin dotted lines are calculated so that they intersect the solid lines at $T = 0.1T_{qb}^s$ (top curve) and $T = 0.2T_{qb}^s$ (bottom curve).

where T_{qb}^s is the solution of:

$$\frac{4}{5}\pi g \left(\frac{T_{qb}^s}{E_F}\right)^2 \ln \frac{E_F}{T_{qb}^s} = 1; \quad (2.7)$$

for large conductances this gives:

$$T_{qb}^s \approx E_F \sqrt{\frac{5}{2\pi g \ln(2\pi g)}}. \quad (2.8)$$

In this regime, the dominant contribution to the singlet correction (2.3) is:

$$\frac{\delta L_{qb}^s}{L_0} = -\frac{1}{5\pi g} \left(\frac{2\pi T\tau}{\hbar}\right)^2 \ln \left(\frac{E_F}{T}\right). \quad (2.9)$$

According to condition (2.6), this expression is applicable if $T_{qb}^s \gg \hbar/2\pi\tau$; this can be satisfied only for large enough conductances. For example at $g = 14$ I find (numerically) $T_{qb}^s \approx 0.11E_F \approx 10\hbar/2\pi\tau$, and at $g = 720$, $T_{qb}^s \approx 0.01E_F \approx 50\hbar/2\pi\tau$; in the latter case, and for larger conductances, Eq. (2.9) can be expected to have a sufficiently large range of validity, while in the former (and generally for small conductances) there is no quasiballistic regime. At temperatures of the order of T_{qb}^s the energy transport becomes truly ballistic in nature, as the dominant processes responsible for the relaxation of the energy current are the inelastic electron-boson collisions and not the electron-impurity collisions. Although the high temperature regime $T \gtrsim T_{qb}^s$ is not considered here, it can be treated within the kinetic equation approach.¹⁸

Fig. 2 shows the relative change of the Lorentz number $\delta L^s/L_0$ as a function of $2\pi T\tau/\hbar$ for five different conductances (solid lines); for the lowest and highest conductances considered, the approximate result (2.9) is also

plotted (dashed lines). In agreement with the above discussion, comparison of the dashed and solid lines shows that Eq. (2.9) deviates significantly from the full expression (2.3) at low conductance, while there is good agreement at high conductance. The intersections between the solid lines and the upper thin dotted curve are at $T = 0.1T_{qb}^s$; it is evident that the region of validity of the quasiballistic approximation grows with the conductance. For all conductances the (negative) correction can be a few percent; in Ref. 13 the Lorentz number was measured²¹ in a 2DEG and found to be slightly smaller than L_0 , in qualitative agreement with the predictions in the present work. However the uncertainties are of the same order of the calculated effect and hence no quantitative comparison is possible.

B. Triplet channel.

For the triplet channel interaction correction, I consider separately, for simplicity, the diffusive and quasiballistic regimes; in the former case the correction is:

$$\frac{\delta L_d^t}{L_0} = \frac{3}{2\pi g} \ln(1 + F_0^\sigma) - \frac{1}{\pi g} \left[1 - \frac{1}{F_0^\sigma} \ln(1 + F_0^\sigma) \right], \quad (2.10)$$

where the first term on the right hand side is again due to the bosonic energy transport, and the second one originates from the interaction-induced energy dependence of the elastic cross section. While the sign of this temperature-independent correction is determined by the sign of the Landau Fermi-liquid constant F_0^σ , its contribution to the total correction δL [Eq. (2.2)] is generally small²² and the overall positive sign of δL at low enough temperatures is determined by δL_d^s . I do not plot separately the contribution (2.10) to δL , since its effect is simply to shift upwards (downwards) the curves in Fig. 1 for $F_0^\sigma > 0$ ($F_0^\sigma < 0$).

In the quasiballistic regime the correction reads:

$$\begin{aligned} \delta L_{qb}^t &= 3\delta L_{qb}^s \left(\frac{F_0^\sigma}{1 + F_0^\sigma}\right)^2 \\ &= -\frac{3}{5\pi g} \left(\frac{2\pi T\tau}{\hbar}\right)^2 \ln \left(\frac{E_F}{T}\right) \left(\frac{F_0^\sigma}{1 + F_0^\sigma}\right)^2 \end{aligned} \quad (2.11)$$

with δL_{qb}^s given in Eq. (2.9). As for the singlet channel, this negative correction originates from the inelastic electron-boson collision, and similarly to the singlet channel correction, the validity of this expression is limited at high temperatures by T_{qb}^t – the equation defining this quantity is obtained by multiplying the left hand side of Eq. (2.7) by $3(F_0^\sigma/1 + F_0^\sigma)^2$. It is evident that, when the quasiballistic approximation is applicable, plotting the sum of Eq. (2.11) and (2.9) would give Fig. 2 with a rescaled vertical axis, as for any value of F_0^σ the triplet channel contribution enhances the singlet channel correction.

III. PARALLEL FIELD DEPENDENCE OF THE LORENTZ NUMBER.

In this section I present the results for the parallel magnetic field dependence of the Lorentz number. The parallel field H affects the electrons by shifting the energy levels by the Zeeman energy

$$E_Z = g_L \mu_B H, \quad (3.1)$$

where g_L is the Lande g-factor and μ_B the Bohr magneton. The Lorentz number depends on H only through this energy and the renormalized Zeeman energy E_Z^* :

$$E_Z^* = \frac{E_Z}{1 + F_0^\sigma}. \quad (3.2)$$

As it is the case for other transport properties (e.g. the magneto-conductivity), it is convenient to consider the deviation ΔL of the Lorentz number from its zero-field value:

$$\Delta L(T, H) = L(T, H) - L(T, 0). \quad (3.3)$$

Once again I address separately the diffusive and quasi-ballistic regimes; in both regimes the system is assumed to be far from the full polarization, i.e. $E_Z \ll E_F$. The derivation of the results can be found in Sec. IV B.

A. Diffusive regime.

For $T \ll \hbar/2\pi\tau$, the field-induced change in the Lorentz number is:

$$\begin{aligned} \frac{\Delta L_d}{L_0} = & -\frac{1}{\pi g} \left(\frac{1}{F_0^\sigma} + \frac{3}{2} \right) \left[I_1 \left(\frac{E_Z}{2\pi T} \right) - I_1 \left(\frac{E_Z^*}{2\pi T} \right) \right] \\ & - \frac{1}{\pi g} \frac{1}{F_0^\sigma} \frac{E_Z}{2\pi T} \left[I_2 \left(\frac{E_Z}{2\pi T} \right) - I_2 \left(\frac{E_Z^*}{2\pi T} \right) \right] \end{aligned} \quad (3.4)$$

with the dimensionless functions I_1 and I_2 defined in Eqs. (4.16)-(4.17). Similarly to Eq. (2.10), the terms with the numerical prefactor $3/2$ are due to the bosonic energy transport, while the remaining ones, proportional to $1/F_0^\sigma$, originate from the energy dependence of the elastic cross section. The structure of this expression as the difference of terms which depend on different energy scales (i.e. E_Z and E_Z^*) can be traced back to the structure of the quantum kinetic equation, in which the bosonic contributions to the collision integral always appear as differences between a soft mode part and a “ghost” part.⁶

In the weak field limit $E_Z, E_Z^* \ll 2\pi T$ Eq. (3.4) becomes:

$$\frac{\Delta L_d}{L_0} \approx -\frac{1}{\pi g} f_d(F_0^\sigma) \left(\frac{E_Z}{2\pi T} \right)^2 \quad (3.5)$$

with

$$f_d(x) = \frac{x(4 + 3x)}{(1 + x)^2}. \quad (3.6)$$

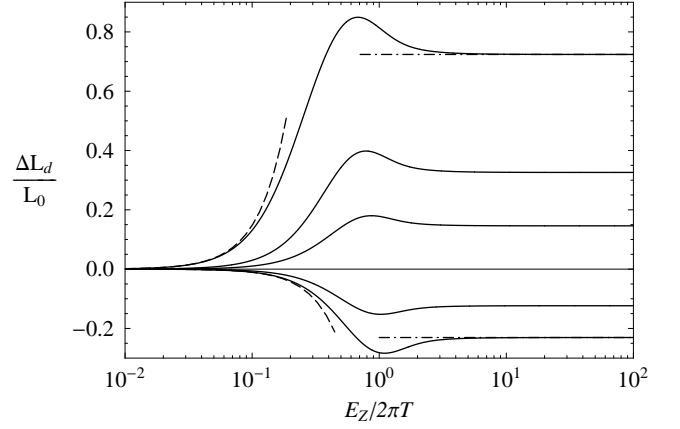


FIG. 3: Relative deviation $\pi g \Delta L_d / L_0$ of the Lorentz number from its zero-field value in the diffusive regime. Solid lines are given by Eq. (3.4) with, from top to bottom: $F_0^\sigma = -0.7, -0.4, -0.2, 0.2$ and 0.4 . For $F_0^\sigma = -0.7$ and 0.4 the approximate expressions (3.5) (dashed lines) and (3.7) (dot-dashed) are also shown for comparison.

In the opposite case $E_Z, E_Z^* \gg 2\pi T$ the approximate formula is:

$$\frac{\Delta L_d}{L_0} \approx -\frac{1}{\pi g} \ln(1 + F_0^\sigma) + \frac{2}{3\pi g} \left[1 - \frac{1}{F_0^\sigma} \ln(1 + F_0^\sigma) \right], \quad (3.7)$$

or equivalently:

$$\Delta L_d \approx -\frac{2}{3} \delta L_d^t \quad (3.8)$$

with δL_d^t given in Eq. (2.10). This result can be explained as follows:¹² in the diffusive regime the correction is dominated by processes with small energy and momentum exchange, and in the strong field the bosonic modes with non-zero spin projection become gapped with the gap given by the Zeeman energy. Therefore the contributions due to these modes must drop out from the total correction to the Lorentz number: this is why the correction (3.7) partially cancels the one given in Eq. (2.10), with the surviving contribution originating from the modes with zero total spin projection.

In Fig. 3 the relative deviation $\Delta L_d / L_0$, multiplied by πg , is plotted as a function of $E_Z / 2\pi T$ for different values of the parameter F_0^σ . At fields such that the Zeeman energy is larger than temperature the deviation becomes quickly field-independent, but near $E_Z \sim 2\pi T$ all the curves are non-monotonic; the presence of peaks is due to the above discussed dependence on the two different energies E_Z and E_Z^* , and through the latter (and the $1/F_0^\sigma$ prefactors) the peaks' positions depend on F_0^σ . The temperature dependence of the deviation at fixed field can also be read from this graph by following the curves from the right (low temperature) to the left (high temperature): the deviation is temperature independent at low temperatures $T \ll E_Z / 2\pi$ and displays

a power-law decay ($\sim T^{-2}$) at high temperatures; again the non-monotonic behavior characterizes the intermediate regime.

B. Quasiballistic regime.

Here I consider the quasiballistic regime $\hbar/2\pi\tau \ll T \ll T_{qb}^t$, with T_{qb}^t defined after Eq. (2.11). In this case I find:

$$\frac{\Delta L_{qb}}{L_0} = -\frac{3}{2\pi g} \left(\frac{2\pi T\tau}{\hbar} \right)^2 \left(\frac{F_0^\sigma}{1+F_0^\sigma} \right)^2 I_3 \left(\frac{E_Z^*}{2\pi T}; \frac{F_0^\sigma}{1+F_0^\sigma} \right) \quad (3.9)$$

with I_3 given in Eq. (4.26). For $E_Z^* \ll 2\pi T$ the result takes the form:

$$\frac{\Delta L_{qb}}{L_0} \approx -\frac{1}{\pi g} f_{qb}(F_0^\sigma) \left(\frac{\tau E_Z}{\hbar} \right)^2, \quad (3.10)$$

where

$$f_{qb}(x) = \left(\frac{x}{1+x} \right)^2 \left[\frac{1+2x+4x^2}{(1+2x)^2} + \frac{2x^2(3+6x+4x^2)}{(1+2x)^3} \ln \left| \frac{x}{1+x} \right| \right], \quad (3.11)$$

while with logarithmic accuracy the large field limit $E_Z^* \gg 2\pi T$ is:

$$\frac{\Delta L_{qb}}{L_0} \approx \frac{2}{5\pi g} \left(\frac{2\pi T\tau}{\hbar} \right)^2 \ln \left(\frac{E_Z}{T} \right) \left(\frac{F_0^\sigma}{1+F_0^\sigma} \right)^2. \quad (3.12)$$

Comparison of Eq. (3.12) with Eq. (2.11) shows that the partial cancelation that was found in the diffusive limit is also realized in the quasiballistic one, but with an important difference: the gapped modes still contribute to the total correction to the Lorentz number because the quasiballistic corrections are dominated by the inelastic scattering with large momentum exchange. The gap therefore excludes the low energy ($E < E_Z$) contributions, but the higher energy ones ($E_Z < E < E_F$) are still relevant.

Fig. 4 shows the field dependence of ΔL_{qb} , Eq. (3.9), by plotting

$$\frac{\Delta L_{qb}}{L_0} \bigg/ \frac{3}{2\pi g} \left(\frac{2\pi T\tau}{\hbar} \right)^2$$

as a function of $E_Z/2\pi T$; for comparison the approximate result (3.12) is also plotted.²³ In the inset the low-field behavior of Eq. (3.9) is compared to Eq. (3.10); except for the case $F_0^\sigma = -0.7$, all curves are non-monotonic. The threshold value F_{th}^σ above which ΔL_{qb} is a non-monotonic function can be found by requiring $f_{qb}(F_{th}^\sigma) = 0$; this gives $F_{th}^\sigma \simeq -0.679$. Although the present results are not valid close to the Stoner instability (see footnote²²), they suggests that as $F_0^\sigma \rightarrow -1$ the relationship between energy and charge transport properties can be qualitatively altered compared to the weakly interacting case.

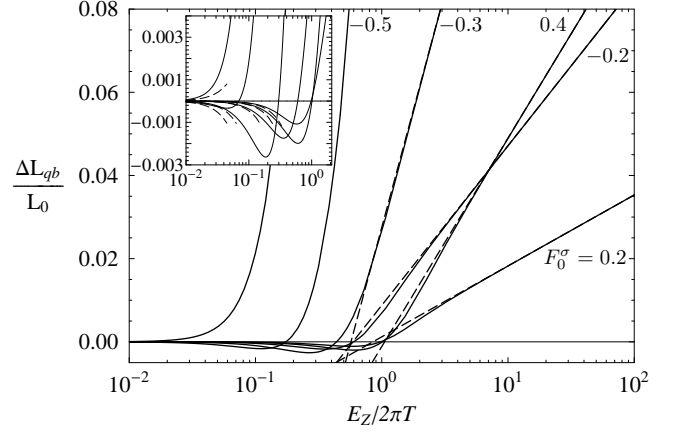


FIG. 4: Relative deviation of the Lorentz number from its zero-field value as a function of the Zeeman energy in the quasiballistic regime (normalized as explained in the text) for different values of the Fermi-liquid parameter, as labeled (the curve partially covered by the inset corresponds to $F_0^\sigma = -0.7$). Solid lines are given by Eq. (3.9), while dashed lines by the approximate expression (3.12). The inset shows details of the low-field regime, with the dashed curves corresponding to Eq. (3.10).

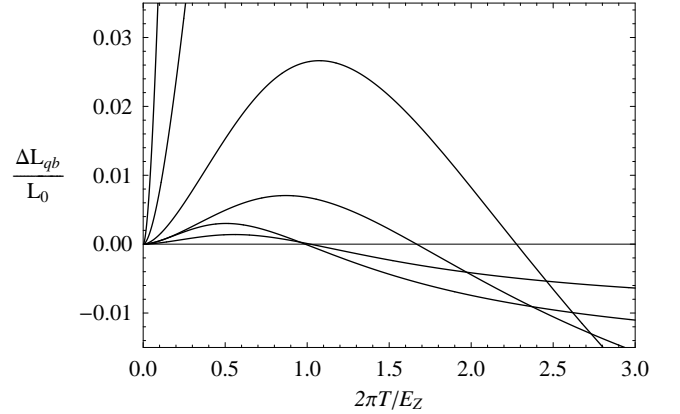


FIG. 5: Relative deviation of the Lorentz number from its zero-field value as a function of the temperature in the quasiballistic regime (normalized as explained in the text) for different values of the Fermi-liquid parameter. Proceeding clockwise near the origin, starting with the steepest curve, the parameters are: $F_0^\sigma = -0.7, -0.5, -0.3, -0.2, 0.4$ and 0.2 .

For completeness, I consider in Fig. 5 the temperature dependence of ΔL_{qb} by plotting

$$\frac{\Delta L_{qb}}{L_0} \bigg/ \frac{3}{2\pi g} \left(\frac{E_Z\tau}{\hbar} \right)^2$$

as a function of $2\pi T/E_Z$ in the low to intermediate temperature regime.

IV. DERIVATION.

This section is devoted to the calculation of the interaction corrections to the Lorentz number using the formalism of Ref. 6. In the absence of the magnetic field, one can use directly the results of that reference, while a generalization is needed for the parallel field case, as discussed in Sec. IV B. From now on, I set $\hbar = 1$.

A. Temperature dependence.

The results presented in Sec. II are a straightforward consequence of the findings of Ref. 6, where it is shown that for two-dimensional systems the thermal conductivity can be written as:

$$\kappa = \kappa_{WF} + \Delta\kappa. \quad (4.1)$$

Here

$$\kappa_{WF} = L_0 \sigma T \quad (4.2)$$

follows the Wiedemann-Franz law with L_0 defined in Eq. (1.1) and the electrical conductivity σ includes the interaction corrections. The additional term $\Delta\kappa = \Delta\kappa^s + 3\Delta\kappa^t$ is given by the sum of the singlet and triplet channel corrections. The former was calculated with logarithmic accuracy:²⁴

$$\begin{aligned} \Delta\kappa_s = & \frac{T}{6} g_1(2\pi T\tau) \ln\left(\frac{v_F k}{T}\right) \\ & - \frac{T}{24} g_2(\pi T\tau) \ln\left(1 + \frac{1}{(2\pi T\tau)^2}\right) \\ & - \frac{2\pi^2}{15} T (T\tau)^2 \ln\left(\frac{E_F}{T}\right) \end{aligned} \quad (4.3)$$

with the functions g_1 and g_2 given by:

$$g_1(x) = \frac{3}{x^2} \left\{ \frac{1}{x} \left[2\psi'(1/x) - x^2 \right] - 2 \right\}, \quad (4.4a)$$

where ψ' is the derivative of the digamma function, and

$$g_2(x) = \frac{26}{15} x^2 + \frac{8}{3} g_1(x) - \frac{5}{3}. \quad (4.4b)$$

Note that $g_1(0) = g_2(0) = 1$, and $g_1(x) \simeq 3/x$ for $x \ll 1$.

Using the definitions (1.2), (4.1) and (4.2) I find at first order in $1/g$:

$$\frac{L(T)}{L_0} = 1 + \frac{\Delta\kappa}{\sigma_D T} \quad (4.5)$$

Then from the definition (2.1) and Eq. (4.3) one arrives at Eq. (2.3). The limiting expressions given in Eqs. (2.5) and (2.9) are found by keeping the leading order contributions to δL^s for small and large $T\tau$ respectively by using

the asymptotic forms of the functions g_1 and g_2 given after Eqs. (4.4). The condition (2.7) is obtained by equating the (absolute value of the) correction (2.9) to the non-interacting Lorentz number L_0 [Eq. (1.1)]; the results are not valid at temperatures higher than T_{qb}^s because in solving the kinetic equation it was assumed that the impurity scattering is the dominant process contributing to the energy relaxation rate – see the discussion at the end of Sec. 6.2 of Ref. 6; however, the kinetic equation itself is still valid.

For the triplet channel, the correction $\Delta\kappa^t$ was calculated in the diffusive and quasiballistic regimes:

$$\Delta\kappa_d^t = -\frac{T}{18} \left[1 - \frac{1}{F_0^\sigma} \ln(1 + F_0^\sigma) \right] + \frac{T}{12} \ln(1 + F_0^\sigma) \quad (4.6)$$

for $T\tau \ll 1/2\pi$, and

$$\Delta\kappa_{qb}^t = -\frac{2\pi^2}{15} T (T\tau)^2 \ln\left(\frac{E_F}{T}\right) \left(\frac{F_0^\sigma}{1 + F_0^\sigma}\right)^2 \quad (4.7)$$

for $T\tau \gg 1/2\pi$. Multiplying Eqs. (4.6)-(4.7) by 3 and using Eq. (4.5) and the definition (2.1) of δL , the results (2.10) and (2.11) are obtained; the factor of 3 arises from the summation over the three projections of the total spin, which in the absence of magnetic field contribute equally to the thermal conductivity.

B. Parallel field dependence.

To obtain the results of Sec. III I give here an extension of the calculations of Ref. 6. As in Eq. (4.1), I separate the thermal conductivity in a part which follows the Wiedemann-Franz law and a correction; both term now depend on the applied parallel magnetic field H , or more precisely on the Zeeman splitting, Eq. (3.1). The term $\kappa_{WF}(T, H)$ is straightforwardly calculated using the results of Ref. 12, so one needs to consider only the correction $\Delta\kappa(T, H)$. As discussed in e.g. Refs. 12 and 17, the singlet and the triplet $L_z = 0$ contributions to κ are not affected by the parallel field [L_z is the projection of the total spin along the field direction]. The effect of the field on the remaining $L_z = \pm 1$ components of the triplet channel correction is to shift the frequency of the interaction propagators; in the language of Ref. 6, the bosonic propagators $\mathcal{L}^\sigma(\omega, \mathbf{q}; \mathbf{n}_1, \mathbf{n}_2; L_z)$ are:

$$\begin{aligned} \mathcal{L}^\sigma(\mathbf{n}_1, \mathbf{n}_2; L_z) = & \Omega_2 \delta(\widehat{\mathbf{n}}_1 \widehat{\mathbf{n}}_2) \mathcal{L}_0(\mathbf{n}_1; L_z) + \\ & \mathcal{L}_0(\mathbf{n}_1; L_z) \mathcal{L}_0(\mathbf{n}_2; L_z) \frac{\left(-i\omega \frac{F_0^\sigma}{1+F_0^\sigma} + \frac{1}{\tau}\right) \mathcal{C}(L_z)}{\mathcal{C}(L_z) - \left(-i\omega \frac{F_0^\sigma}{1+F_0^\sigma} + \frac{1}{\tau}\right)}, \end{aligned} \quad (4.8)$$

and the corresponding (triplet) “ghost” propagators $\mathcal{L}^g(\omega, \mathbf{q}; \mathbf{n}_1, \mathbf{n}_2; L_z)$ are:²⁵

$$\begin{aligned} \mathcal{L}^g(\mathbf{n}_1, \mathbf{n}_2; L_z) = & \Omega_2 \delta(\widehat{\mathbf{n}}_1 \widehat{\mathbf{n}}_2) \mathcal{L}_0(\mathbf{n}_1; L_z) + \\ & \mathcal{L}_0(\mathbf{n}_1; L_z) \mathcal{L}_0(\mathbf{n}_2; L_z) \frac{\frac{1}{\tau} \mathcal{C}(L_z)}{\mathcal{C}(L_z) - \frac{1}{\tau}}, \end{aligned} \quad (4.9)$$

where

$$\mathcal{L}_0(\mathbf{n}_i; L_z) = \frac{1}{-i(\omega - L_z E_Z^*) + i\mathbf{v}_i \cdot \mathbf{q} + 1/\tau}, \quad (4.10)$$

$$\mathcal{C}(L_z) = \sqrt{(-i(\omega - L_z E_Z^*) + 1/\tau)^2 + (v_F q)^2}.$$

In the above formulas I dropped the variables ω , \mathbf{q} for compactness, $\mathbf{v}_i = v_F \mathbf{n}_i$ and E_Z^* is the Zeeman energy renormalized by the interactions, Eq. (3.2).

The calculation of the field-dependent thermal conductivity proceeds now as in Ref. 6: the evaluation of the transport coefficients can be reduced to integrals over the frequency ω whose integrands consist of a distribution function part times a kernel part $K(\omega)$; the latter is found after integration over the momentum \mathbf{q} and summation over the total spin projections. The field-dependent kernels can be found using the expressions (4.8)-(4.9) instead of their zero-field counterparts; below I calculate explicitly the correction $\Delta\kappa_m$ to the thermal magnetoconductivity $\kappa_m(T, H) = \kappa(T, H) - \kappa(T, 0)$. In other words, I want to write $\kappa(T, H)$ in the form (4.1) and define $\Delta\kappa_m$ as:

$$\Delta\kappa_m(T, H) = \Delta\kappa(T, H) - \Delta\kappa(T, 0), \quad (4.11)$$

where $\Delta\kappa(T, 0)$ is the correction considered in the previous section. To calculate $\Delta\kappa_m$ I introduce for each kernel K the corresponding kernel difference $\Delta K = K(H) - K(0)$ between the kernel calculated with and without the field. As in the preceding section, I consider separately the diffusive and quasiballistic regimes.

1. Diffusive regime.

In the diffusive regime, the two main contributions to $\Delta\kappa$ come from the energy dependence of the elastic cross section and from the bosonic energy transport. Writing

$$\Delta\kappa_{m,d} = \delta\kappa_{el,m} + (\kappa_m^\sigma - \kappa_m^g), \quad (4.12)$$

the two terms are given by [cf. Eqs. (6.11b) and (6.36b) of Ref. 6]:

$$\delta\kappa_{el,m} = -2 \frac{\sigma_D}{e^2 T} \int d\omega \Delta\mathcal{E}(\omega) \left[\frac{\omega^3}{12} \frac{\partial N_F}{\partial \omega} \right] \quad (4.13a)$$

and

$$\kappa_m^\sigma - \kappa_m^g = \frac{\sigma_D}{e^2 T} \int d\omega \Delta\mathcal{B}^0(\omega) \left[\frac{\omega^3}{4} \frac{\partial N_F}{\partial \omega} \right] \quad (4.13b)$$

with the kernels

$$\Delta\mathcal{E} = -\frac{e^2}{\sigma_D 2\pi^2 \omega^2} \frac{1}{F_0'} \left[\omega \ln \left| \frac{\omega^2 - E_Z^2}{\omega^2 - E_Z^{*2}} \right| + E_Z \ln \left| \frac{\omega + E_Z}{\omega - E_Z} \cdot \frac{\omega - E_Z^*}{\omega + E_Z^*} \right| \right] \quad (4.14a)$$

and

$$\Delta\mathcal{B}^0 = \frac{e^2}{\sigma_D 2\pi^2 \omega} \ln \left| \frac{\omega^2 - E_Z^2}{\omega^2 - E_Z^{*2}} \right|. \quad (4.14b)$$

As discussed above, these kernels are found by substituting the expressions (4.8)-(4.9) for the propagators into the definitions of \mathcal{E} and \mathcal{B}^0 given in Eqs. (6.9) and (6.36d) of Ref. 6.²⁶ Substituting Eqs. (4.14) into Eqs. (4.13) [and by a change of variable $\omega \rightarrow 2\pi T\omega$] I get:

$$\delta\kappa_{el,m} = -\frac{T}{6} \frac{1}{F_0'} \left\{ I_1 \left(\frac{E_Z}{2\pi T} \right) - I_1 \left(\frac{E_Z^*}{2\pi T} \right) + \frac{E_Z}{2\pi T} \left[I_2 \left(\frac{E_Z}{2\pi T} \right) - I_2 \left(\frac{E_Z^*}{2\pi T} \right) \right] \right\} \quad (4.15a)$$

and

$$\kappa_m^\sigma - \kappa_m^g = -\frac{T}{4} \left[I_1 \left(\frac{E_Z}{2\pi T} \right) - I_1 \left(\frac{E_Z^*}{2\pi T} \right) \right], \quad (4.15b)$$

where I introduced the dimensionless functions:

$$I_1(E) = \pi \int d\omega \frac{\omega^2}{\sinh^2 \pi \omega} \ln \left| 1 - \frac{E^2}{\omega^2} \right| \quad (4.16)$$

and

$$I_2(E) = \pi \int d\omega \frac{\omega}{\sinh^2 \pi \omega} \ln \left| \frac{1 + \omega/E}{1 - \omega/E} \right|. \quad (4.17)$$

Eq. (4.16) can be identically rewritten as:

$$I_1(E) = 2E^2 - 4 \sum_{n=1}^{\infty} n \left[\ln \left(1 + (E/n)^2 \right) - \frac{(E/n)^2}{1 + (E/n)^2} \right] \quad (4.18)$$

which is useful to obtain the $E \ll 1$ expansion, and as:

$$I_1(E) = \frac{2}{3} \ln |E| + C + \pi \int d\omega \frac{\omega^2}{\sinh^2 \pi \omega} \ln \left| 1 - \frac{\omega^2}{E^2} \right| \quad (4.19)$$

which gives the $E \gg 1$ asymptotic behavior; the constant C appearing above is:

$$C \equiv -2\pi \int d\omega \frac{\omega^2}{\sinh^2 \pi \omega} \ln |\omega| = \frac{2}{3} \left(\gamma + \ln 2\pi - \frac{3}{2} \right) - \frac{4}{\pi^2} \zeta'(2) \simeq 0.99, \quad (4.20)$$

where $\gamma \simeq 0.577$ is the Euler constant and $\zeta'(2) \simeq -0.938$ is the derivative of the zeta function evaluated at 2. As for Eq. (4.17), in the given form the large E limit can be readily obtained [$I_2(E) = 2/3E + \dots$], while to find the small E limit I rewrite it as:

$$I_2(E) = \pi \operatorname{sgn} E - 4E + 4 \sum_{n=1}^{\infty} \operatorname{Im} \left[\ln \left(1 + i \frac{E}{n} \right) - \frac{1}{1 - iE/n} \right]. \quad (4.21)$$

Note also the identity [primes indicate derivatives]:

$$I_1'(E) + EI_2'(E) = 0 \quad (4.22)$$

which enables to verify that the correction $\delta\kappa_{el,m}$ of Eq. (4.15a) vanishes in the limit $F_0^\sigma \rightarrow 0$, as expected.

Using Eqs. (4.15), together with the definitions (4.1), (4.11) and (3.3), and dropping terms of higher order in $1/g$, one arrives at Eq. (3.4). The approximate expressions (3.5)-(3.7) follow from Eqs. (4.18)-(4.21).

2. Quasiballistic regime.

In the quasiballistic regime, the correction $\Delta\kappa$ is determined by the inelastic electron-boson collisions and it can be written as [cf. Eq. (6.39a) of Ref. 6]:

$$\Delta\kappa_{m,qb} = \frac{\sigma_D}{e^2 T} \int d\omega \Delta\mathcal{B}^1(\omega) \left[\frac{\omega^3}{4} \frac{\partial N_P}{\partial \omega} \right] \quad (4.23)$$

with

$$\Delta\mathcal{B}^1 = \frac{e^2}{\sigma_D 2\pi^2} \tau^2 \omega \left(\frac{F_0^\sigma}{1 + F_0^\sigma} \right)^2 J \left(\frac{E_Z^*}{\omega}; \frac{F_0^\sigma}{1 + F_0^\sigma} \right), \quad (4.24a)$$

$$J(E; F) = -2 \sum_{L_z = \pm 1} \left\{ \ln |1 - L_z E| \frac{(1 - L_z E)^2}{(1 - L_z E)^2 - F^2} - \ln |F| F^2 \left[\frac{1}{(1 - L_z E)^2 - F^2} - \frac{1}{1 - F^2} \right] \right\}. \quad (4.24b)$$

Some details on the derivation of this kernel are given in Appendix B. Substitution of Eqs. (4.24) into Eq. (4.23) results in

$$\Delta\kappa_{m,qb} = -\frac{T}{4} (2\pi T \tau)^2 \left(\frac{F_0^\sigma}{1 + F_0^\sigma} \right)^2 I_3 \left(\frac{E_Z^*}{2\pi T}; \frac{F_0^\sigma}{1 + F_0^\sigma} \right) \quad (4.25)$$

with the dimensionless function I_3 defined as:

$$I_3(E; F) = \pi \int d\omega \frac{\omega^4}{\sinh^2 \pi \omega} J \left(\frac{E}{\omega}; F \right). \quad (4.26)$$

The large and small E limits of I_3 can be found by keeping the leading order terms in the expansion of the function $J(E; F)$.²⁷ In this way I obtain:

$$I_3(E; F) \approx -\frac{4}{15} \ln |E| \quad (4.27)$$

for $E \gg 1$, and

$$I_3(E; F) \approx \frac{2}{3} E^2 \left[\frac{1 + 3F^2}{(1 - F^2)^2} + \frac{F^2(3 + F^2)}{(1 - F^2)^3} \ln F^2 \right] \quad (4.28)$$

for $E \ll 1$. Knowing $\Delta\kappa_{m,qb}$ and the approximate formulas (4.27)-(4.28), proceeding as in the previous subsection finally leads to Eq. (3.9), (3.10) and (3.12).

V. CONCLUSIONS.

The quantum kinetic equation approach is a powerful method to investigate the effects of the electron-electron interactions on transport in disordered metals^{5,6,12} and open quantum dots.¹¹ Using this approach I considered the temperature and parallel magnetic field dependence of the Lorentz number in two-dimensional disordered metals.

Three regimes can be distinguished as the temperature increases: diffusive, quasiballistic and truly ballistic. In the low-temperature diffusive regime, the Lorentz number is enhanced above its Drude value due to the energy transported by neutral bosonic modes that describe the interacting electron-hole pairs, see Eq. (2.5) and Fig. 1. At intermediate temperatures (the quasiballistic regime) the Lorentz number is suppressed by the inelastic electron-boson collision, Eq. (2.9) and Fig. 2, with the crossover between the two regimes described by Eq. (2.3). The effect of the interaction in the triplet channel is given in Eqs. (2.10) and (2.11) for the diffusive and quasiballistic regimes respectively.

If a magnetic field is applied parallel to the two-dimensional metal, the Zeeman splitting of the electronic energies affects the transport properties; in particular in the diffusive regime the deviation of the Lorentz number from its zero-field value displays a non-monotonic dependence on the ratio between the Zeeman energy and the temperature, see Eq. (3.4) and Fig. 3. Finally, as discussed in Sec. III B, in the quasiballistic regime the deviation can be either a monotonic or non-monotonic function of both temperature and Zeeman energy depending on the value of the Fermi-liquid parameter.

Acknowledgments

I am grateful to I. Aleiner and Y. Ahmadian for reading and commenting on the manuscript. Financial support through the J.A. Krumhansl Postdoctoral Fellowship is acknowledged.

APPENDIX A: PARALLEL FIELD DEPENDENCE OF THE SPECIFIC HEAT.

In this appendix I calculate for completeness the correction δC_V to the specific heat in the presence of a parallel magnetic field; the general expression for δC_V is:⁶

$$\delta C_V = \frac{\partial}{\partial T} (u^\sigma - u^g), \quad (A1)$$

where the energy densities u^α are given by:

$$u^\alpha = \int d\omega \omega b^\alpha(\omega) N_P(\omega). \quad (A2)$$

Here $N_P(\omega)$ is the Planck distribution and $b^\alpha(\omega)$ are the bosonic densities of states. In the presence of the parallel

field I find:

$$b^\sigma(\omega; H) - b^g(\omega; H) = -\frac{\text{Re}}{2\pi} \sum_{L_z} \int \frac{d^2 q}{(2\pi)^2} \left[\frac{F_0^\sigma}{1 + F_0^\sigma} \left(\frac{1}{\mathcal{C}(L_z) - b} - \frac{1}{\mathcal{C}(L_z)} \right) - \frac{-i(\omega - L_z E_Z^*) + 1/\tau}{\mathcal{C}(L_z)} \left(\frac{1}{\mathcal{C}(L_z) - b} - \frac{1}{\mathcal{C}(L_z) - 1/\tau} \right) \right] \quad (\text{A3})$$

with \mathcal{C} defined in Eq. (4.10) and

$$b = -i\omega \frac{F_0^\sigma}{1 + F_0^\sigma} + \frac{1}{\tau} \quad (\text{A4})$$

Performing the integration and the summation I arrive at:

$$\begin{aligned} & \left(b^\sigma(\omega; H) - b^g(\omega; H) \right) - \left(b^\sigma(\omega; 0) - b^g(\omega; 0) \right) = \\ & -\frac{1}{8\pi^2 D} \left[\frac{1}{1 + F_0^\sigma} \ln \left| 1 - \frac{E_Z^2}{\omega^2} \right| - \ln \left| 1 - \frac{E_Z^{*2}}{\omega^2} \right| \right] \\ & + \tau\pi(|\omega| - E_Z^*) \left(\theta(E_Z^2 - \omega^2) - \theta(E_Z^{*2} - \omega^2) \right) \\ & - \left(\frac{F_0^\sigma}{1 + F_0^\sigma} \right)^2 \pi\tau|\omega| \theta(E_Z^2 - \omega^2). \end{aligned} \quad (\text{A5})$$

Next, I substitute this result into Eq. (A2) and then into Eq. (A1); taking the temperature derivative and rescaling the frequency $[\omega \rightarrow 2\pi T\omega]$ I finally obtain:

$$\begin{aligned} & \delta C_V(H) - \delta C_V(0) = \\ & \frac{1}{4} \frac{T}{D} \left[I_1 \left(\frac{E_Z^*}{2\pi T} \right) - \frac{1}{1 + F_0^\sigma} I_1 \left(\frac{E_Z}{2\pi T} \right) \right] \\ & + \frac{1}{\pi} \frac{T}{D} (T\tau) \left[\left(\frac{F_0^\sigma}{1 + F_0^\sigma} \right)^2 f_3 \left(\frac{E_Z}{2T} \right) + f_3 \left(\frac{E_Z^*}{2T} \right) \right. \\ & \left. - f_3 \left(\frac{E_Z}{2T} \right) - \left(\frac{E_Z^*}{2T} \right) \left(f_2 \left(\frac{E_Z^*}{2T} \right) - f_2 \left(\frac{E_Z}{2T} \right) \right) \right] \end{aligned} \quad (\text{A6})$$

with I_1 defined in Eq. (4.16) and

$$f_n(z) = \int_0^z d\omega \frac{\omega^n}{\sinh^2 \omega}. \quad (\text{A7})$$

The functions f_n can be given in terms of polylogarithms as:

$$\begin{aligned} f_2(z) &= \frac{\pi^2}{6} + 2z \log(1 - e^{-2z}) - \text{Li}_2(e^{-2z}) \\ &+ z^2(1 - \coth z) \\ f_3(z) &= \frac{3}{2} \zeta(3) + 3z^2 \log(1 - e^{-2z}) - 3z \text{Li}_2(e^{-2z}) \\ &- \frac{3}{2} \text{Li}_3(e^{-2z}) + z^3(1 - \coth z) \end{aligned} \quad (\text{A8})$$

The second line in Eq. (A6) is the correction to the specific heat in the diffusive limit, while the last two lines become dominant in the quasiballistic limit. In this limit and for weak interaction $|F_0^\sigma| \ll 1$, Eq. (A6) reproduces the result of Ref. 19.

APPENDIX B: DERIVATION OF THE KERNEL $\Delta\mathcal{B}^1$.

In this appendix I briefly outline how to derive the kernel $\Delta\mathcal{B}^1$ given in Eq. (4.24a) starting from the results of Sec. 6.2 of Ref. 6. There, the exact (at linear order in ∇T) solution of the kinetic equation was given; this solution is unaffected by the parallel field. In the quasiballistic regime, the main corrections to the thermal conductivity were found to originate from the term in the bosonic distribution function defined as δN^1 and which can be neglected in the diffusive limit; these corrections are given in Eqs. (6.35c) and (6.36c) of Ref. 6. Performing the angular integrations in those equations results in Eqs. (6.38a) and (6.38b) respectively; by repeating those calculations using the propagators in Eqs. (4.8)-(4.9) a similar result is obtained in the presence of the parallel field. The explicit expression is simply found by redefining some of the quantities appearing on the right hand sides of Eqs. (6.38) as follows: for the function \mathcal{C} , I substitute the function $\mathcal{C}(L_z)$ given in Eq. (4.10); the parameter b' is now:

$$b'(L_z) = -i(\omega - L_z E_Z^*) + \frac{1}{\tau};$$

all the other quantities, namely b [Eq. (A4)] and

$$\tilde{N} = v_F \tau \frac{\omega^2}{T} \frac{\partial N_P}{\partial \omega} \frac{F_0^\sigma}{1 + F_0^\sigma}$$

are unchanged.

To arrive at the kernel $\Delta\mathcal{B}^1$, the sum over L_z need to be performed, along with the remaining integral over the magnitude of the momentum \mathbf{q} . This integral was performed in Ref. 6 with logarithmic accuracy; in the present case however both the infrared divergence (due to the long-range part of the Coulomb interaction) and the ultraviolet divergence are absent – the latter because I consider here the kernel difference, the former because the triplet channel interaction is short-range – and I proceed in a different manner. By rescaling all the dimensional quantities ω , E_Z^* , $1/\tau$ and $v_F \mathbf{q}$ by the temperature T , I find that the contributions due to Eq. (6.38a) are smaller than those of Eq. (6.38b) by $1/T\tau$ and can therefore be neglected. Similarly, to find the leading term in the (Laurent) expansion over $1/T\tau$ one can set $1/\tau \rightarrow 0$ in Eq. (6.38b). Following this procedure, I get:

$$\begin{aligned} & \int \frac{d^2 q}{(2\pi)^2} \int \frac{d\mathbf{n}_1 d\mathbf{n}_2}{\Omega_2} n_{1\mu} \text{Re} \{ \mathcal{L}^\sigma(L_z) \delta_\nu N_{12}^1 \} \\ & \simeq -\delta_{\mu\nu} \frac{\tau\omega}{8\pi v_F^2} \frac{F_0^\sigma}{1 + F_0^\sigma} \tilde{N} \int_0^\infty dp \left[\text{Re} \frac{1}{\sqrt{p - (1 - L_z E_Z^*/\omega)^2}} \right. \\ & \quad \times \text{Re} \frac{1}{\sqrt{p - (1 - L_z E_Z^*/\omega)^2} - i \frac{F_0^\sigma}{1 + F_0^\sigma}} \\ & \quad \left. \times \left(1 - \frac{(1 - L_z E_Z^*/\omega)^2}{p} \right) \right], \end{aligned}$$

where in terms of the original variables $p = (v_F q / \omega)^2$, and the approximate equality indicates that I am neglecting higher order terms in $1/T\tau$. The integral over p is logarithmically divergent, but the difference between the above expression and the similar one at zero parallel field is finite. The exact integration of this difference gives finally the expression for the kernel $\Delta\mathcal{B}^1$ given in Eqs. (4.24).

APPENDIX C: PARALLEL FIELD DEPENDENCE OF THE ELECTRICAL CONDUCTIVITY.

The aim of this appendix is to compare the present approach to the one of Ref. 12 for the calculation of the parallel field magneto-conductivity $\Delta\sigma$, which is given by:

$$\Delta\sigma = \sigma_D \int d\omega \left[\Delta\mathcal{E}(\omega) + \Delta\mathcal{S}^{el}(\omega) \right] \frac{\partial}{\partial\omega} \left[\omega N_P(\omega) \right]. \quad (\text{C1})$$

This formula follows from Eq. (6.8) of Ref. 6, and the kernels are defined in the subsequent Eq. (6.9).

In the diffusive limit only $\Delta\mathcal{E}$ is relevant, since the kernel \mathcal{S}^{el} gives contributions smaller by the factor $T\tau \ll 1$, as discussed in Ref. 6; it is straightforward to verify that substituting Eq. (4.14a) into Eq. (C1), the diffusive limit result of Ref. 12 is recovered. Viceversa, it can be shown that in the quasiballistic limit the kernel \mathcal{E} can be neglected since larger corrections are due to \mathcal{S}^{el} – this can be done for example by rescaling all dimensionful quantities by the temperature, as described in Appendix B. Proceeding as detailed there, i.e. dropping higher order terms in $1/T\tau$, and introducing the shorthand notation:

$$s(a|b) \equiv \text{sgn}(a) - \text{sgn}(b),$$

I obtain for the kernel $\Delta\mathcal{S}^{el}$ in the quasiballistic regime:

$$\Delta\mathcal{S}^{el} = \Delta\mathcal{S}^{11} + \Delta\mathcal{S}^{12} \quad (\text{C2a})$$

$$\begin{aligned} \Delta\mathcal{S}^{11} \simeq & \frac{e^2}{\sigma_D 2\pi^2} \frac{\tau}{\omega} \frac{\pi}{2} \sum_{L_z = \pm 1} \left[s(\omega - L_z E_Z | \omega - L_z E_Z^*) \right. \\ & \times (\omega - L_z E_Z^*) - \omega \frac{F_0^\sigma}{1 + F_0^\sigma} s(\omega - L_z E_Z | \omega) \left. \right] \end{aligned} \quad (\text{C2b})$$

$$\begin{aligned} \Delta\mathcal{S}^{12} \simeq & \frac{e^2}{\sigma_D 2\pi^2} \frac{\tau}{\omega} \frac{\pi}{2} \sum_{L_z = \pm 1} \left[s(\omega - L_z E_Z | \omega - L_z E_Z^*) \right. \\ & \times (\omega - L_z E_Z^*) \frac{2\omega F_0^\sigma}{\omega(1 + 2F_0^\sigma) - L_z E_Z} \\ & \left. - \omega \frac{F_0^\sigma}{1 + F_0^\sigma} s(\omega - E_Z | \omega) \right]. \end{aligned} \quad (\text{C2c})$$

Eqs. (C1) and (C2) lead to the quasiballistic limit result of Ref. 12; in particular, the sum of the first terms in square brackets gives the contribution denoted there by K_2 , while the remaining terms give the K_1 contribution. Here I point out that in the low-field limit $E_Z, E_Z^* \ll 2T$, the quasiballistic magneto-conductance is given by:

$$\Delta\sigma \approx \frac{e^2}{\pi} \frac{2F_0^\sigma}{1 + F_0^\sigma} T\tau \frac{1}{3} \left(\frac{E_Z}{2T} \right)^2 f(F_0^\sigma), \quad (\text{C3})$$

where

$$\begin{aligned} f(z) = 1 - \frac{z}{1+z} \left[1 + \frac{1}{2} \frac{1}{1+2z} + \frac{1}{(1+2z)^2} \right. \\ \left. - \frac{2(1+z) \ln 2(1+z)}{(1+2z)^3} \right]. \end{aligned} \quad (\text{C4})$$

This expression corrects the wrong definition of $f(z)$ given after Eq. (14) of Ref. 12.²⁸ While the difference between the two definitions is numerically small (less than 4%) for $-0.57 \lesssim F_0^\sigma \lesssim 0.14$, it grows rapidly outside this parameter range, and use of Eq. (C4) rather than its counterpart of Ref. 12 may be important in a comparison between theory and experiment.

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- ²⁰ The applicability of Eqs. (2.3) and (2.5) is limited to values of the gas parameter far from the Wigner crystal instability, i.e. $r_s \lesssim 37$ – see also Sec. IIIB of Ref. 5.
- ²¹ The experiment of Ref. 13 was presumably performed in the quasiballistic regime, since the conductivity showed a linear in temperature behavior (cf. Ref. 5).
- ²² The correction (2.10) becomes large near the Stoner instability $F_0^\sigma = -1$; in this regime, however, approximating the Landau Fermi-liquid parameter F^σ by its (momentum independent) average over the Fermi surface F_0^σ severely restricts the temperature range over which the present results can be applied to $T/E_F \ll (1 + F_0^\sigma)^2$, see Ref. 5. Therefore I do not consider this regime.
- ²³ This equation gives the slope of the dashed straight lines; a subleading constant, calculated numerically, has been added to each line.
- ²⁴ Here an algebraic mistakes in Ref. 6 is corrected; this correction leads to different numerical factors which, however, do not modify qualitatively the original results.
- ²⁵ The ghosts corresponding to the singlet channel are, of course, unchanged.
- ²⁶ Alternatively, one can use directly the diffusive limit expressions for these kernels, Eqs. (6.22) and (6.24) of Ref. 6 with the appropriately modified diffusive limit propagators, which are obtained by shifting the frequency $\omega \rightarrow \omega - L_z E_Z^*$.
- ²⁷ The calculation of the subleading terms is more involved for I_3 compared to the expansion of I_1 or I_2 and I do not attempt it here.
- ²⁸ The erroneous definition in Ref. 12 is probably the consequence of an algebraic mistake. As a further check, I have verified numerically that Eq. (C4) is indeed the right formula.